USING PORTFOLIO ASSIGNMENTS TO ASSESS STUDENTS' MATHEMATICAL THINKING

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A model in use at Prospect Hill Academy in Cambridge, Massachusetts, provides a skills-based portfolio assessment for mathematics classes.

Writing in mathematics can improve procedural knowledge and communication skills and may also help students better understand and then remember problems (Cai, Jakabcsin, and Lane 1996; Drake and Amspaugh 1994; Johanning 2000; Zupancic and Ishii 2002). Student writing can also allow teachers to recognize and diagnose the nature of students' conceptual problems (Drake and Amspaugh 1994). Perhaps of most interest, researchers have found that writing-to-learn activities "can have a small, positive impact on conventional measure of academic achievement" (Bangert-Drowns, Hurley, and Wilkinson 2004, p. 29). The majority of mathematics teachers know that they ought to include some writing assignments in their instructional plans, but the challenge of covering the

curriculum and the time required to plan and institute a learning activity of this nature are high hurdles. However, the research is compelling: Mathematics teachers should be including writing activities consistently in instructional plans. We have found that writing activities are most effective when we use an explicit and well-designed model and when we work in teams of teachers to support one another.

Our purpose is to show how teachers have implemented a portfolio assessment model at an urban charter school with impressive results. We will avoid duplicating what is already presented in NCTM's assessment handbook for grades 9–12 (Bush and Greer 2001). A well-prepared section on portfolio assessments (pp. 81–89) describes some of the decisions that teachers have to make to use a portfolio assignment in class. The handbook provides a variety of artifacts related to the creation of a portfolio that asks students to collect and purposefully organize work to demonstrate mastery of course goals, along with pages of student work reproduced in reduced form (pp. 85–89).

THE PORTFOLIO DESIGN REQUIRES STUDENTS TO DEMONSTRATE MASTERY OF ESSENTIAL UNIT SKILLS, EXPLAIN PROCEDURES, AND COMMUNICATE CONCEPTUAL UNDERSTANDING.

We offer an alternative model for a portfolio assignment and show how we teach students to create portfolios successfully at Prospect Hill Academy Charter School in Cambridge, Massachusetts. The Prospect Hill Academy portfolio design requires students to demonstrate mastery of essential unit skills, explain procedures, and communicate conceptual understanding. All students are asked to complete such a portfolio at least once per quarter. Whereas most portfolio approaches require students to select, assemble, and reflect on previously completed work, the Prospect Hill Academy portfolio requires students to create new work and is an integral component of unit design, two important differences from the model presented by Bush and Greer (2001).

Effective use of a complex assessment of this nature is challenging and time-consuming. Convincing teachers to commit to and sustain an effort to use portfolio assessments in their classes will require more than research—after all, traditional tests are quicker and easier to deal with. The success we have witnessed at Prospect Hill Academy is what keeps us using portfolios. Our students have developed the ability to read and write about mathematics through portfolios and have submitted more self-directed, higher-quality work on other assignments. Our efforts have contributed to Prospect Hill Academy's becoming one of the top-performing schools on the state-mandated tenth-grade mathematics test, with 95 percent of all students achieving proficiency in 2008 and 2009. We are confident that our portfolio assessments have played a major role in these successes.

THE PORTFOLIO ASSESSMENT MODEL

The format of the portfolio is always the same. It includes the following items:

- *Cover and table of contents*
- *Cover letter*—Students summarize what they learned during the unit and identify what was new knowledge, what was review material, and what questions remain.
- *Vocabulary*—For each term selected by teachers at the beginning of the unit, students write the mathematical definition; a paraphrase of the definition in their own words; an example (which can also be a drawing); "something that looks like an example but is not"; and an explanation of how the two "examples" fulfill or fail to fulfill the definition.
- *Skill descriptions*—For each teacher-identified skill objective in the unit, students write a step-by-step description of how to "do" the skill and provide worked examples and an explanation of when and where to apply the skill.
- *Unit questions*—Unit questions vary in type but require students to describe conceptual understanding.
- *Reflection*—Students reflect on their classroom participation, behavior, and quality of work during the unit and identify specific areas for improvement.

Long before the start of each unit, we start thinking about the portfolio assignment—specifically, what skills, vocabulary, and concepts are most important for students to learn.

PLANNING THE UNIT PORTFOLIO

Identifying Skill Objectives

The most important step is to identify the unit's critical learning objectives. Using the school's standards-based curriculum maps, teams of two or more

Definition Template for Portfolio Entries

1. State the term.
2. State the mathematical definition of the term you learned in class.
3. Explain the meaning of the term in your own words. Next year (or on the final) you should be able to look at this explanation and understand what the word means even if you have not thought about the material for a while.
4. Show an example of the term and show something that looks like an example but is not. Explain what the difference between the two examples is. You need to explain carefully what detail or attribute makes the "not example" what you claim.
5. Show another way that you can think about this term or something that will help you remember the concept. You may want to show a picture, graph, diagram, or drawing that will help you remember the big idea.

Fig. 1 The template for vocabulary includes providing a nonexample, an important skill.

teachers select the three to five most important skills for students to master by the end of the unit. Such prioritizing also helps teachers focus unit plans, manage instructional time, and concentrate student attention on a few essential skill outcomes. When describing skills, we are careful to use action verbs and student-friendly language, and we ensure that the skills can be assessed. Following are some sample skill outcomes:

- "I can write the equation of a line passing through two given points."
- "Given a function family relationship, I can translate between a table, a graph, and an equation."
- "I can solve a system of equations in two variables using four different methods."
- "I can determine the area of a complex shape by using the area formulas for simpler shapes."

Selecting Unit Vocabulary Terms

Unit-specific and frequently occurring mathematical terms should be intentionally and directly taught. Examples are *solve*, *function*, *evaluate*, *simplify*, *factor*, *independent variable*, *quadratic equation*, *hypotenuse*, *exponential function*, and *system of equations*. Use of a template (like that in **fig. 1**) helps support students in their mathematical writing.

Designing Unit Questions

Unit questions are designed to promote deeper exploration of mathematical concepts and ideas. Following are some examples of such questions:

How to Write a Skill Description

Skill descriptions are records of your mastery of the skill. The five parts to write are described below.

1. Skill title
A title gives you a short way to remember what the skill is [a few words].

2. Skill description
This is a one- to three-sentence description of what the skill is and what it is good for. This description should be short enough that you can reread it quickly and decide if you could make use of the skill to solve a problem that you are working on.

3. Examples
Use one or two challenging and relevant examples.

4. Application
You need to explain how, if you were looking at a word problem, you would know that this skill could help you solve the problem.

5. Description of the process
Describe the process in a list (step 1, step 2, etc.) or in paragraph form. Be sure to explain the purpose of each step, such as, "I did this step because at this point in the process I needed to isolate x and the way to do that was to …."

Fig. 2 The template for skill descriptions gives students a flexible framework for composing their responses.

- "Show the general form of a quadratic equation and explain what characteristics of the function are 'easy' and which are 'hard' to see in this form."
- "What are three different ways of describing the slope of a line, and how are they related?"
- "How are exponential functions and logarithmic functions different? How are they similar?"
- "When I have a collection of data (table or graph), how do I decide what kind of function will be the best model (e.g., linear or exponential)?"
- "When I see an algebraic representation of a function, how do I determine whether I need to use the chain rule to determine the derivative?"

IMPLEMENTING THE UNIT PORTFOLIO

Setting Up Students (and Teachers) for Success

It is important to assign the portfolio in sections over time. We have noticed that students produced higher quality work and that they learned more when they thought about skills over multiple weeks, an observation supported by research (Rohrer and Taylor 2006). For example, if the first topic in the first quarter was right-triangle trigonometry, the students would begin working on the skill description ("Knowing one side and an angle of a right triangle, I can determine a missing

side") after about one week. Writing the skill entry then becomes the students' work for a few days. We often have them start this skill description in class one day and include work on it as part of their homework for the next two or three days.

To help students organize their work and their thinking, we provide them with a template for writing a skill description (see **fig. 2**). Distributing a checklist that asks, "Do you have a _____?" for each piece of a skill description is also beneficial. Not surprisingly, students initially find the skill descriptions very difficult to write, particularly if they have rarely been asked to write about their mathematics before. Moreover, our textbooks often do not have a set of step-by-step descriptions for how to complete an algorithm, and thus writing this description and justifying it is a substantial challenge for students.

To guide students in learning how to meet these challenges, we might use graphic organizers to help them meet the demands of this section of the portfolio (see **fig. 3**). Similarly, we suggest that students follow these basic principles:

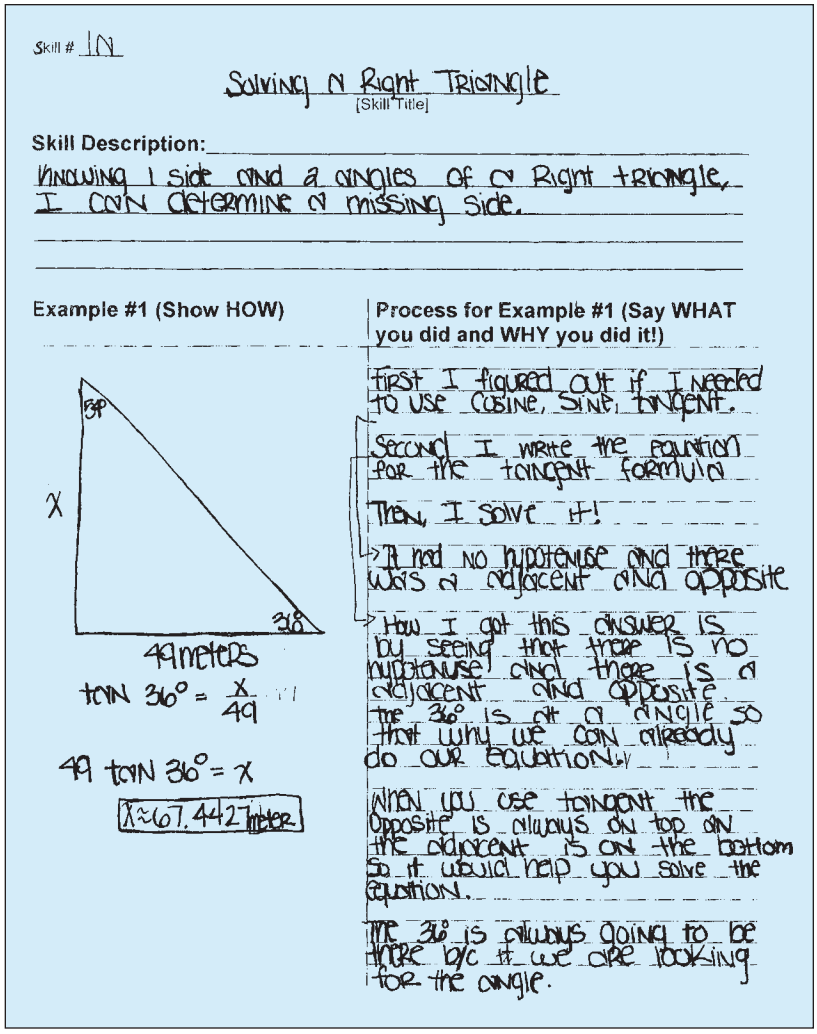


Fig. 3 An organizer such as this one, developed by our colleague Kathryn Jenks, can help students formulate their portfolio responses.

- Always write more than you think is necessary.
- Always include examples.
- Use appropriate mathematical language, *including specified vocabulary.*

This type of work requires a level of scaffolding significantly above and beyond what a teacher might generally provide for more traditional assignments.

We consider a first attempt a draft, mirroring the process for effective writing. We give students questions and suggestions for improvement and then require them to revise their drafts during class time or as part of their homework. The portfolio design provides authentic opportunities for differentiation, and teachers can work closely with students to achieve the optimal level of challenge. For example, the teacher may require that a student work on more complex examples or include a "general" process rather than one focused on a particular problem.

We repeat this procedure with all the skill descriptions and unit questions. Students receive precise feedback on specific skills and knowledge, feedback that allows them to refine their techniques and understanding, and teachers are able to target misunderstandings quickly. Whenever possible, teachers at Prospect Hill Academy work collaboratively to assess student portfolio assignments, primarily to ensure common performance standards but also to provide mutual support and encouragement to sustain the process in the face of pedagogical and organizational (time) challenges.

Figure 3 shows a graphic organizer used as scaffolding for a ninth-grade student's writing. **Figure 4** shows the work of a tenth-grade student during the first quarter of a second-year algebra course, when right-triangle geometry was being reviewed. Neither student demonstrates a complete justification of the procedure at hand, but each does show the type of explanation that we might expect and that is typical of the type of student work we see at these grade levels.

At the end of each unit, students assemble their portfolios, write their reflections, and submit them for a final grade. For most students, the portfolio serves as a highly useful resource for review for our more conventional unit and midterm assessments. To avoid being swamped with grading at the end of a unit, we try to provide feedback and evaluation regularly.

GIVING FEEDBACK TO STUDENTS

The most significant improvements in the quality of student work—and, consequently, in learning—came when we provided substantial feedback to students and asked them for revisions. Teachers' comments should "identify what has been done well and what still needs improvement, and give

guidance on how to make that improvement" (Black et al. 2003, p. 49) as well as give students an immediate opportunity to follow up on the suggestions. Teachers at Prospect Hill Academy have created a guide for teaching writing across the curriculum based on research and practice (Bowman 2008). Here are the guide's most helpful ideas about responding to student work.

Give Positive Feedback

Students need to know what works in their writing if they are to repeat successful strategies and make them a permanent part of their repertoire as writers. Our feedback begins with the good work and correct mathematics and explicitly states why those parts are of a high standard. Subsequently, when asking for improvements, we can refer to the qualities in the good work.

Write in Complete, Detailed Sentences

Cryptic comments such as "more detail needed," and "justification?" will be incompletely understood by most students, who will wonder what kind of detail is required and where and what the teacher means by "justification." Symbols and abbreviations such as *"awk"* and "?" are likewise confusing. Correcting student work is clearly not advisable. The more specific and concrete the comments, the more helpful they will be to student writers.

Writing comments in the margins of students' work was typically not productive; they rarely, if ever, seemed to help students improve the quality of their work. Rather, longer, more detailed comments with a focus on a few key areas, where the items requiring improvement are ranked in order of importance, elicited the substantive revisions that we hoped for rather than the superficial editing that we had previously been seeing.

Use a Respectful Tone

Even in the face of fatigue and frustration, it is important to address students respectfully, as the junior scholars and colleagues they are (Bowman 2008).

Maintain Consistency in Feedback

We always make our comments on a standard form (see **fig. 5**) so that the students do not have to decipher different types of teacher commentary. Another reason we use this form rather than write on student work is that doing so makes us, the teachers, reflect on what we want to say to our students. Reflection allows us to help them prioritize their work and to temper our comments.

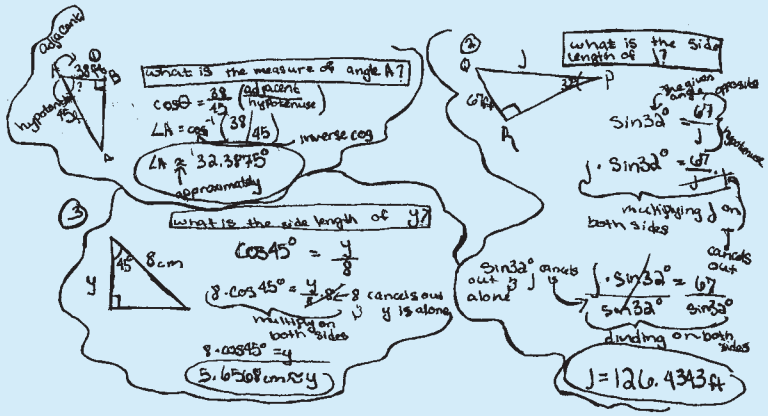
CONCLUSION

Ultimately, adopting a portfolio assessment of this nature requires a serious commitment of time and energy, good advance planning, and, ideally, a colleague or two with whom to collaborate. Our portfolio assessment design forces teachers to identify key unit skills, seek convincing evidence of understanding, and seriously engage students in their learning. Through the portfolio generation process, we believe that we are able to convince our students of three critical facts:

1. We hold high expectations of them and believe that they can meet these expectations with appropriate instruction and support.
2. We really care about their learning and want them to learn, grow, and succeed as students.
3. Hard work, more than talent, is the key to success.

Skill 1 (one) [uno]

Skill description: Demonstrate that you can use sin/cos/tan and inverse sin/cos/tan to find angles and sides in triangles.



Process: *To find the side of a right triangle with a given side and angle, you have to do the following steps:*

- Note the sides you have (adjacent, hypotenuse, and opposite)
- Determine to use cos/tan/sin
- Plug in the given angle, for example, if the angle was 45° and the angles given agreed with cosine, then I would write cos45°
- Then I would write the cosine of the sides, given *x*
- Then you get *x* alone
- If *x* was the numerator, then you multiply the denominator by cos/tan/sin(angle); and if *x* was the denominator, then you divide the numerator by cos/tan/sin(angle)

To find the angle of a right triangle with two sides, you do the following:

- Note the sides you have (adjacent, hypotenuse, and opposite)
- Determine to use cos/tan/sin
- Write the cos/tan/sin for the triangle. In this case, you should use inverse cos/sin/tan
- Then you plug step 3 into a calculator (multiply inverse cos/sin/tan by the sides of the triangle)

Fig. 4 This skill description was written by a tenth-grade second-year algebra student.

These are probably the most fundamental beliefs we can instill in our students to promote high achievement, and we believe that we have seen results. Our students' performance on the state mathematics test is impressive, few of our students fail mathematics classes, and, perhaps most significant, the majority of our students take advanced mathematics courses in their upper high school years. On a broader level, our graduation rate is very high, and almost all our students are admitted to college before graduation.

We do not believe that our portfolio assignments are responsible for all such achievement. However, we do believe that these complex written assessments contribute to our students' mathematical and overall success, above and beyond the requirements of the state's standardized achievement test.

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Comments on Portfolio 3 Draft

One thing this portfolio did well was: You did a great job talking about all the conjectures for special parallelograms and explaining them in your own words.

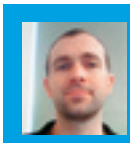
Focus on the following thing(s) for revision:

1. In your explanation in example 1, go step by step so that you can show how you worked through the problem.

2. Make sure that you state all the conjectures that go along with your parallelogram diagrams, not just tell the characteristics of each one.

	+ (Good)	(Change)
Example 1 Problem	You did a good job picking a problem that had several different parts to it and solved it correctly.	You might want to add a small problem that includes the rules of midsegments (either a triangle or a trapezoid).
Example 1 Process	You did a good job explaining the conjectures/rules that you used in order to solve this problem.	You need to go step by step to explain how you solved each of these missing letters. It would be helpful to show what angle you started with and what conjectures went with it before moving on to another one. This would make things easier to follow in your explanation.

Fig. 5 A standard form makes it easier for students to decode teacher comments.

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